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THE LIMITING MOISTURE PROFILE DURING INFILTRATION IN TO A HOMOGENEOUS SOIL

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The present paper deals with a quasilinear second order parabolic equation describing an unsteady one-dimensional infiltration and investigates the time asymptotic of the solution of the problem of formation of moisture saturation profile when the infiltration starts at the surface. The existence of a limiting profile expanding with a constant velocity is proved and estimates are given for the speed of approach to this profile with increasing time, when the soil has unlimited capacity. An estimate of the speed of approach to the steady (homogeneous) distribution is also given for the soil of limited capacity.

During the infiltration into a homogeneous soil, moisture $u(t, x)$ of the soil being a function of time t and of depth x of the layer (the X-axis is directed downwards), satisfies an equation of the type

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[D(u)\frac{\partial u}{\partial x}\right] - \frac{\partial K(u)}{\partial x} \tag{1}$$

$$D(u) > 0, \quad K(u) > 0, \quad D'(u) > 0, \quad K'(u) > 0, \quad K''(u) \geqslant \mu > 0 \text{ when } (u \geqslant u_0 > 0)$$

Taking into account initial moisture distribution in the soil and infiltration on the surface of the ground, we obtain the following boundary condition:

$$u(t, 0) = u_1 \quad (t > 0), \qquad u(0, x) = u_0(x) \quad (0 \leqslant x < \infty)$$

$$u_0 \leqslant u_0(x) \leqslant u_1 \quad (\lim u_0(x) = u_0) \text{ when } x \to +\infty) \tag{2}$$

Here $u_1 = 1$ denotes the moisture corresponding to full saturation of soil on the earth surface.

In the presence of ground water at the depth $x = X$, our boundary condition assumes the form

$$u(t, 0) = u_1, \; u(t, X) = u_1, \; u(0, x) = u_0(x)$$

$$0 \leqslant x \leqslant X, \; u_0 \leqslant u_0(x) \leqslant u_1 \tag{3}$$

The problem of determination of the limiting moisture profile during infiltration into the soil i.e. investigation of asymptotic behavior of solutions of boundary value problems (1), (2) and (1), (3) as $t \to \infty$, is of great practical value in the problems of irrigation. A number of foreign authors [1 to 4] investigated the physical and partly mathematical aspect of this problem, and they have put forward an assertion which was either based on physical considerations, or was arrived at intuitively. It stated that after sufficient time the moisture profile assumes some permanent form, which then moves downwards with a constant velocity without further change. Il'in and Oleinik have investigated in [5] the asymptotic behavior of solutions of Cauchy's problem for Eq. (1) with $D(u) \equiv$ const, encountered in gas dynamics.

Methods given in the present paper extend to the case $D(u) \not\equiv$ const, applied to boundary value problems (1), (2) and (1), (3) (Cauchy's problem has, in this case, no physical meaning).

Let the coefficients of (1) and the boundary functions satisfy the conditions of existence and uniqueness theorems, the latter being bounded together with the derivatives of solutions of boundary value problems (1), (2) and (1), (3) (see [6]).

We shall denote by $U(x - At + C)$ $(A > 0)$ a simple wave solution of (1), satisfying the condition

$$U(-\infty) = u_1, \qquad U(+\infty) = u_0 \tag{4}$$

Integration of (1) together with (4), yields

$$x - At + C = \int_{u_2}^{U} \frac{D(u)\,du}{[K'[u_0 + \Theta(u - u_0)] - A](u - u_0)} \qquad (0 < \Theta(u) < 1) \tag{5}$$

It is easy to see that when $u_0 < u_2 < u_1$ and $K''(u) \geqslant \mu > 0$, then a simple wave solution satisfying (4) exists, is a monotonously decreasing function since

$$\frac{\partial U}{\partial x} = \frac{[K'[u_0 + \Theta(U - u_0)] - A](U - u_0)}{D(U)} < 0 \tag{6}$$

for any finite values of x and t, and is defined with accuracy of up to the displacement C along the X-axis. Velocity A of the parallel displacement of the wave is given by

$$A = \frac{K(u_1) - K(u_0)}{u_1 - u_0}$$

Theorem 1. Let $u(t, x)$ be a solution of the problem (1), (2). If the initial function $u_0(x)$ satisfies, at all $x \geqslant 0$, the inequality

$$u_0(x) - u_0 \leqslant M_1 e^{-\gamma_1 x}, \qquad \gamma_1 > \frac{|K'(u_0) - A|}{D(u_0)} \tag{7}$$

(where M_1 is a constant), then such constants $M > 0$, C_0 and $\beta > 0$ independent of the solution $u(t, x)$ exists, that the inequality

$$|u(t, x) - U(x - At + C_0)| \leqslant M e^{-\beta t} \tag{8}$$

holds.

Proof. Let us perform the following change of variables: $t' = t$, $x' = x - At$ and return to the former variables t and x. Then, the boundary value problem (1), (2) reduces to

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[D(u)\frac{\partial u}{\partial x}\right] - \frac{\partial K^\circ(u)}{\partial x} \tag{9}$$

$$u(0, x) = u_0(x) \quad (0 \leqslant x < \infty), \qquad u(t, -At) = u_1 \quad (t \geqslant 0) \tag{10}$$

$$K^\circ(u) = K(u) - Au + C^\circ$$

Constant C° is chosen so that $K^\circ(u_0) = K^\circ(u_1) = 0$. Solution of the problem (9), (10) is defined in the region $P\{t \geqslant 0, -At \leqslant x < \infty\}$, bounded by $\Gamma\{t \geqslant 0, x = -At\}$.

Simple wave solutions $U(x - At + C)$ of (1) satisfying condition (4) now become stationary solutions $U(x + C)$ of

$$\frac{d}{dx}\left[D(u)\frac{du}{dx}\right] = \frac{dK^\circ(u)}{dx} \tag{11}$$

also satisfying (4).

The basic method of obtaining the inequality (8) utilizes a generalized maximum principle formulated as follows.

Lemma. Let a function $u(t, x)$ be defined and continuous in the region P

$$u(t, x) \geqslant 0 \text{ on } \Gamma; \; u(t, x) \geqslant M(t)\,[1 + |x|]$$

where $M(t)$ is a continuous function. If the inequality

$$L(u) = a(t, x)\frac{\partial^2 u}{\partial x^2} + b(t, x)\frac{\partial u}{\partial x} + c(t, x)\frac{\partial u}{\partial t} + d(t, x)\,u \leqslant 0, \qquad (x, t) \in P$$

where $a(t, x)$, $b(t, x)$ $c(t, x)$ and $d(t, x)$ are bounded in P and $d(t, x) \leqslant 0$ while $c(t, x) \leqslant -k_0 < 0$ holds, then $u(t, x) \geqslant 0$ in P.

This Lemma is completely analogous to Lemma 1 in [5], and we shall use it to prove the following well known property of solutions of (9).

If $u_1(t, x)$ and $u_2(t, x)$ are two solutions of (9) and $u_1(t, x)|_\Gamma \leqslant u_2(t, x)|_\Gamma$, then $u_1(t, x) \leqslant u_2(t, x)$ in P. In particular, we have by virtue of this property

$$u_0 \leqslant u(t, x) \leqslant u_1 \tag{12}$$

Let us now consider two simple wave solutions $U_1(x + \lambda_1 t + C_1)$ and $U_2(x - \lambda_2 t - C_2)$, $(C_1, C_2 > 0)$ of (9), each satisfying relevant conditions

$$U_1(-\infty) = u_1, \quad U_1(+\infty) = u_0 - \varepsilon; \quad U_2(-\infty) = u_1 + \varepsilon, \quad U_2(+\infty) = u_0$$

where the wave velocities $\lambda_1 > 0$ and $\lambda_2 > 0$ are given by

$$\lambda_1 = \frac{K^\circ(u_1) - K^\circ(u_0 - \varepsilon)}{u_1 - u_0 + \varepsilon}, \qquad \lambda_2 = \frac{K^\circ(u_1 + \varepsilon) - K^\circ(u_0)}{u_1 + \varepsilon - u_0}$$

We shall assume the $\varepsilon > 0$ is sufficiently small to satisfy

$$\lambda_2 \leqslant D(u_0)\,\gamma_1 - K^{\circ\prime}(u_0), \; \lambda_1 < A \tag{13}$$

Let us estimate the difference $U_2(x - \lambda_2 t - C_2) - u_0$. By analogy with (6), we have

$$\frac{dU_2}{ds} = \frac{\partial U_2}{\partial x} = \frac{[K^{\circ\prime}(u_0 + \Theta(U_2 - u_0)) - \lambda_2](U_2 - u_0)}{D(U_2)}$$

$$(0 < \Theta(U_2) < 1, \; s = x - \lambda_2 t - C_2)$$

Obviously

$$\frac{K^{\circ\prime}(u_0) - \lambda_2}{D(u_0)} \leqslant \frac{K^{\circ\prime}(u_0 + \Theta(U_2 - u_0)) - \lambda_2}{D(U_2)} < 0$$

Let us put

$$\frac{K^{\circ\prime}(u_0) - \lambda_2}{D(u_0)} = -m_2 \quad (m_2 < \gamma_1)$$

Thus

$$-m_2(U_2 - u_0) \leqslant \frac{dU_2}{ds} \leqslant 0, \qquad \text{or} \qquad -m_2 ds \leqslant d\ln(U_2 - u_0) \leqslant 0$$

This yields

$$U_2 - u_0 \leqslant e^{-m_2 s} < K_2 e^{-m_2(x - \lambda_2 t)} \tag{14}$$

The inequality

$$u_1 - U_1 \leqslant K_1 e^{m_1(x + \lambda_1 t)} \qquad \left(m_1 = \frac{K^{\circ\prime}(u_1) + \lambda_1}{D(u_0)}\right) \tag{15}$$

is derived in an exactly analogous manner.

When C_1 and C_2 increase, initial value of $U_1(x + C_1)$ decreases while that of $U_2(x - C_2)$ increases, and by (13) and (14) we have

$$\frac{u_0(x) - u_0}{U_2(x - C_2) - u_0} \leqslant M_0 < \infty$$

i.e. the initial function $u_0(x)$ converges to u_0 not slower than $U_2(x - C_2)$. Consequently, constants C_1 and C_2 can be chosen so that the inequalities

$$U_1(x + C_1) \leqslant u_0(x) \leqslant U_2(x - C_2) \qquad (0 \leqslant x < \infty)$$

hold. But then we have

$$U_1(x + \lambda_1 t + C_1) \leqslant u(t, x) \leqslant U_2(x - \lambda_2 t - C_2) \quad (x, t) \in P \tag{16}$$

From (12) and (16) we can obtain the following properties of the solution $u(t, x)$ of the boundary value problem (9) and (10)

(1) $\lim\limits_{x \to +\infty} u(t, x) = u_0$ when $t \in [0, T]$ (T is any finite number)

(2) $\lim\limits_{t \to +\infty} \dfrac{\partial u}{\partial x} = 0$ when $t \in [0, T]$

(3) $\lim_{t\to\infty} \frac{\partial u(t, -At)}{\partial x} = 0$

(4) At any finite t, the following integral exists

$$\int_x^\infty [u(t, x) - u_0]\, dx \leqslant \int_x^\infty [U_2(x - \lambda_2 t - C_2) - u_0]\, dx$$

The first property follows directly from (12) and (16). The second property can be proved using inequalities in a manner completely analogous to that used in Lemma 3 of [5]. To prove the third property we subtract u_1 from all parts of the inequality

$$U_1^0(x + \lambda_1 t + C_1) \leqslant u(t,x) \leqslant u_1$$

Using (15) we have

$$\left| \int_{-At}^{x} \frac{\partial u}{\partial x}\, dx \right| = |u(t, x) - u_1| \leqslant K_1 e^{m_1 (x+\lambda_1 t)}$$

By arguments analogous to those given in the proof of Lemma 5 in [4] we find, that since $|\partial u/\partial x| < M = \text{const}$, the latter inequality implies that

$$\left| \frac{\partial u(t, x)}{\partial x} \right| < K_0 e^{1/2\, m_1 (x+\lambda_1 t)} \tag{17}$$

and this, in turn, gives the final result

$$\left. \frac{\partial u}{\partial x} \right|_{x=-At} \to 0 \quad \text{as} \quad t \to \infty$$

Assertion (4) follows from the inequality (12) and existence (by (14) of the integral

$$\int_x^\infty [U_2(x - \lambda_2 t - C_2) - u_0]\, dx$$

Properties (1) to (4) of the solution of (9), (10) can be used to prove the existence of a limit to the function

$$J(t) = \int_{-At}^{0} [u(t, x) - u_1]\, dx + \int_0^\infty [u(t, x) - u_0]\, dx$$

Indeed

$$\frac{\partial}{\partial t}\left[\int_{-At}^{0} [u(t, x) - u_1]\, dx + \int_0^\infty [u(t, x) - u_0]\, dx \right] = \int_{-At}^{\infty} \frac{\partial u(t, x)}{\partial t}\, dx =$$

$$= \int_{-At}^{\infty} \frac{\partial}{\partial x} \left\{ \left[D(u) \frac{\partial u}{\partial x} \right] - K^\circ(u) \right\} dx = D(u_1) \left. \frac{\partial u(t, x)}{\partial x} \right|_{x=-At}$$

Since

$$K^\circ(u_0) = K^\circ[u(t, -At)] = K^\circ(u_1) = 0. \left| \frac{\partial}{\partial x} u(t, -At) \right| \leqslant K_0 e^{-m_1 (A-\lambda_1)t}$$

then

$$\lim_{t\to\infty} \frac{\partial}{\partial t} J(t) = 0$$

Obviously the function

$$\frac{d}{dz} J\left(\frac{1-z}{z}\right) \qquad \text{when } z = \frac{1}{t+1} \in (0, 1] \quad \left(\left.\frac{dJ}{dz}\right|_{z=0} = 0 \right)$$

is continuous. This specifically implies that

$$\lim_{t\to\infty} J(t) = \lim_{z\to 0} J\left(\frac{1-z}{z}\right) = B$$

exists.

Let us also estimate the rate of convergence of $J(t)$ to the constant B

$$|J(t) - B| = \left| J\left(\frac{1-z}{z}\right) - B \right| \leqslant \left| \frac{dJ}{dz} \right| z = \left| \frac{dJ}{dt} \right| \left| \frac{dt}{dz} \right| \frac{1}{(t+1)} = \left| \frac{dJ}{dt} \right| (t+1)$$

thus we have

$$|J(t) - B| \leqslant K_1 (t+1) e^{-m_1(A-\lambda_1)t} \tag{18}$$

Next we shall establish the convergence, uniform in t, of the solution $u(t, x)$ to u_0 as $x \to +\infty$. Putting

$$\alpha = -\frac{\max |K^{\circ\prime}(u)|}{D(u_0)} \geqslant 0$$

we choose the constant M appearing in

$$y(t, x) = \int_x^\infty \{Me^{-\alpha x} - [u(t, x) - u_0]\}\, dx$$

sufficiently large to make $y(0, x) \geqslant 0$. This is possible since $\gamma_1 > \alpha$. Integration of (9) from x to ∞ yields

$$D(u)\frac{\partial u}{\partial x} - K^\circ(u) + \frac{\partial}{\partial t}\int_x^\infty u(t, x)\, dx = 0 \tag{19}$$

Let $v(x) = Me^{-\alpha x} + u_0$. Obviously

$$\begin{gathered} D(u)\frac{\partial v}{\partial x} - K^\circ(v) = -Me^{-\alpha x}[D\alpha + K^{\circ\prime}(\Theta)] \leqslant 0 \\ u_0 \leqslant \Theta \leqslant u_0 + Me^{-\alpha x}, \quad |K^{\circ\prime}(u_0)| > |K^{\circ\prime}(\Theta)| \end{gathered} \tag{20}$$

Subtracting (19) from (20) and taking into account that

$$-(v-u) = \frac{\partial}{\partial x}\left(\int_x^\infty (v(t, x) - u(t, x))\, dx\right) = \frac{\partial y}{\partial x}$$

we obtain the following inequality

$$D(u)\frac{\partial^2 y}{\partial x^2} - K^{\circ\prime}(\Theta)\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} \leqslant 0$$

Since the function $y(t, x)$ satisfies the conditions of the Lemma, we have

$$\int_x^\infty [u(t, x) - u_0]\, dx \leqslant \frac{1}{\alpha} Me^{-\alpha x}$$

from which, the uniform in t convergence of the solution $u(t, x)$ to u_0 as $x \to +\infty$, follows.

We shall now consider the function

$$z(t, x) = \int_x^\infty [u(t, x) - U(x + C_0)]\, dx$$

where $U(x + C_0)$ is a stationary solution of (11) satisfying the condition (4) with constant C_0 defined by

$$\int_{-\infty}^0 [U(x + C_0) - u_1]\, dx + \int_0^\infty [U(x + C_0) - u_0]\, dx = B$$

Such a constant exists by virtue of a monotonous dependence of $U(x + C_0)$ on C.

We shall prove that when $t \to \infty$, the magnitude $|z(t, x)|$ can be made smaller than any given $\varepsilon > 0$. Then the inequality $|z(t, x)| < \varepsilon$ when $t \geqslant T$ and boundedness of the derivative, with respect to x, of the integrand function yields the estimate (see [5])

$$|u(t, x) - U(x + C_0)| < K_0^\circ \sqrt{\varepsilon} \tag{21}$$

for $t \geqslant T$ where K_0° is independent of t.

Let us integrate each of the identities

$$\frac{\partial}{\partial x}\left[D(u(t, x))\frac{\partial u(t, x)}{\partial x}\right] - \frac{\partial u(t, x)}{\partial t} - \frac{\partial K^\circ[u(t, x)]}{\partial x} \equiv 0$$

$$\frac{\partial}{\partial x}\left[D(U(x + C_0))\frac{\partial U(x + C_0)}{\partial x}\right] - \frac{\partial U(x + C_0)}{\partial t} - \frac{\partial K^\circ[U(x + C_0)]}{\partial x} \equiv 0$$

from x to ∞, subtracting subsequently the second resulting identity from the first.

Taking into account the fact that

$$u - U = -\frac{\partial}{\partial x}\int_x^\infty [u(t, x) - U(x + C_0)]\,dx$$

we obtain

$$D(u)\frac{\partial^2 z}{\partial x^2} - \left[D'(\Theta_1)\frac{\partial U}{\partial x} - K^{\circ\prime}(\Theta_2)\right]\frac{\partial z}{\partial x} - \frac{\partial z}{\partial t} \equiv 0$$

where Θ_1 and Θ_2 are mean values between $u(t, x)$ and $U(x + C_0)$. Let us denote

$$D(u) = D,\ \left[D'(\Theta_1)\frac{\partial U}{\partial x} - K^{\circ\prime}(\Theta_2)\right] = B,\ L(y) = D\frac{\partial^2 y}{\partial x^2} + B\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t}$$

Let us choose a sufficiently large N such that when $|x| > N$ the inequality $B < -\nu_0 < 0$ holds (this is possible by virtue of the uniform convergence of $u(t, x)$ to u_0). Then

$$L(e^{-\alpha \exp \lambda x}) = \alpha\lambda e^{-\alpha \exp \lambda x} e^{\lambda x}[\lambda D(\alpha \exp \lambda x - 1) + B] < 0$$

for $|x| < N$, $\alpha = k^{-1}\exp(-\lambda N)$ $(k \geqslant 1)$ and with sufficiently large λ;

$$L(e^{-\alpha x}) = \alpha[\alpha D - B]e^{-\alpha x} < 0$$

for $x > N$ and with sufficiently large k.

Clearly, we can construct a function $Q(x)$ continuous together with its second order derivative, coinciding with the function $\varphi(x) \equiv \exp \lambda x$ when $x \geqslant N$, smoothly becoming linear $\varphi(x) \equiv x$ when $x > N$ and such that

$$L(e^{-\alpha Q(x)}) \leqslant -\delta e^{-\alpha Q(x)} < 0$$

Function

$$W(t, x) = M_1 e^{-\alpha Q(x) - \beta t} + \varepsilon \pm z(t, x)$$

is nonnegative on the boundary Γ of P when α and β are fixed and M_1 is sufficiently large

Indeed, when $x \geqslant N$ and $t \geqslant T$, we have $|z(0, x)| < \varepsilon$ and

$$|z(t, -At)| = |I(t) - B| < \varepsilon$$

Consequently, when M_1 is sufficiently large, we have

$$W(0, x) = M_1 e^{-\alpha Q(x)} + \varepsilon \pm z(0, x) \tag{22}$$

$$W(t, -At) = M_1 e^{-\alpha Q(-At) - \beta t} + \varepsilon \pm z(t, -At) \geqslant 0 \tag{23}$$

Also

$$L(W) = L(M_1 e^{-\alpha Q(x) - \beta t}) + (L \pm z + \varepsilon) < (-\delta + \beta)e^{-\alpha Q(x) - \beta t} < 0 \text{ when } \beta < \delta$$

Thus function $W(t, x)$ satisfies the conditions of the Lemma and

$$\left|\int_x^\infty [u(t, x) - U(x + C_0)]\,dx\right| < M_1 e^{-\alpha Q(x) - \beta t} + \varepsilon \tag{24}$$

If, taking into account the inequalities (7) and (18) we put

$$\alpha = \min\left\{\frac{1}{k \exp \lambda N},\ \gamma_1\right\}$$

and choose $\beta \leqslant \delta$ small enough to ensure that $\alpha A - \beta > -m_1(A - \lambda_1)$, then we can put $\varepsilon = 0$ in (22), (23) and (24). From (21) we obtain

$$|u(t, x) - U(x + C_0)| < M e^{-1/2\alpha Q(x) - 1/2\beta t}$$

where M is a constant independent of t.

Returning now to the former variables x and t, we obtain the proof of our Theorem.

Theorem 2. Let $u(t, x)$ be a solution of the boundary value problem (1), (3). There exist such constants M and β, that the inequality

$$|u(t, x) - u_1| < M e^{-\beta t}$$

holds.

Proof. We shall write (1) as

$$D(u)\,\partial^2 u/\partial x^2 + [D'(u)\,\partial u/\partial x - K^{\circ\prime}(u)]\,\partial u/\partial x - \partial u/\partial t = 0$$

and put

$$\left[D'(u)\frac{\partial u}{\partial x} - K^{\circ\prime}(u)\right] = B,\quad D(u) = D$$

Function

$$W(t, x) = M_1 e^{-\alpha \exp \lambda x - \beta t} \pm [u(t, x) - u_1]$$

is nonnegative on the boundary Γ_1 of the region $R\{t \geqslant 0,\ 0 \leqslant x \leqslant X\}$ when α is fixed, and M_1 is sufficiently large. Since at sufficiently small α we have

$$L(W) = D\frac{\partial^2 W}{\partial x^2} + B\frac{\partial W}{\partial x} - \frac{\partial W}{\partial t} = \alpha\lambda \exp(-\alpha e^{\lambda x} - \beta t)\, e^{\lambda x}\left[\lambda D(\alpha e^{\lambda x} - 1) - B\right] < 0 \quad (25)$$

in the region R, function $W(t, x)$ cannot attain a negative minimum in R as at the point of negative minimum we have $\partial^2 W/\partial x^2 \geqslant 0$, $\partial W/\partial x = 0$ and $\partial W/\partial t \leqslant 0$, which implies $L(W) > 0$ which in turn contradicts (25).

Thus $W(t, x) \geqslant 0$ in R and it follows that

$$|u(t, x) - u_1| \leqslant M_1 e^{-\alpha \exp \lambda x - \beta t}$$

The assertion of Theorem 2 is obvious.

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